**Nuclear Physics**

Now we’ll step back a little bit, and look at the nucleus.

Table

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**The Strong Nuclear Force and Nuclear Stability**

Nuclei are identified in the following notation: , where A is the atomic number, and Z the proton number. For instance, in the periodic table, carbon is listed as having 6 protons and 6 neutrons. So its nucleus would be represented as . This is indeed the most cohesive (stable) form of carbon. But there are other types of carbon nuclei, called ‘isotopes’, which have a different number of neutrons. For instance a carbon atom with 7 neutrons would be written . All nuclei in the periodic table have isotopes, which are merely nuclei with same # of protons, but different # of neutrons.

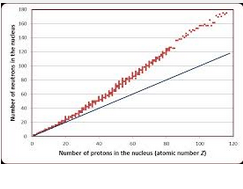
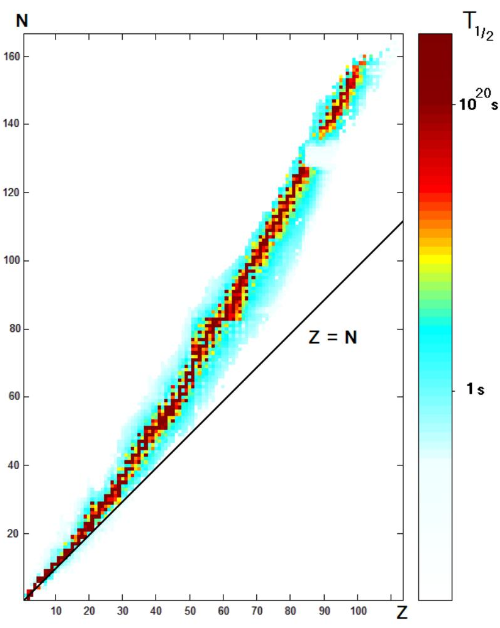
*A* being proportional to volume in some sense, the nuclear radius goes as R0A1/3. R0 is approximately 1.2 fm. So a typical nucleus is about 10fm in radius, compared to the atomic radius of about 10pm; so the nucleus is about 1000 times smaller than the atom itself. A nucleus is composed of nucleons, i.e. protons and neutrons.

One might wonder why nuclei are even held together since the protons all exert repulsive forces on each other. The answer is that there is another force between protons (and neutrons), called the strong nuclear force. This force is attractive, and as its name suggests, very strong, stronger than the electric force. And it is this force which binds the nucleus together. This force as a few peculiar characteristics:

1. This force only acts between nucleons (well hadrons, really, but whatever)
2. Also, the nuclear force acts strongest between pairs of protons and neutrons, and especially pairs of pairs. Thus, the α particle (He nucleus) is an exceptionally cohesive object.
3. It saturates, meaning that a single nucleon can only experience a force with a few other nucleons simultaneously. This is unlike the electric force, where a proton can feel the force of an infinite number of other protons simultaneously.
4. It exponentially decays after a distance of about 1fm, in marked contrast to the electric force which never exponentially decays. Thus, the strong nuclear force is stronger than the repulsive electric force only within a distance of around 1fm. And this is why nuclei are not much bigger than that size.

This is evinced by the fact that the binding energy per nucleon is roughly constant as we keep adding nucleons to a nucleus. Otherwise, the binding energy would increase as the nucleon would interact with more and more other nucleons. This is unlike the electric force, where a proton can exert an electric force simultaneously with all other protons. Also, the nuclear force favors binding of pairs of protons/neutrons with opposite spins, and especially pairs of pairs. Thus, the α particle is an exceptionally stable object.

Given these, absent the electric force, a nucleus would prefer to have the constant ratio N = Z, so that there would be pairs. And for small nuclei, this is indeed the case. But as Z increases, the repulsive force between the protons increases greatly, as the protons feel the repulsive electric force of every other proton in the nucleus. However they feel the attractive strong nuclear force of only its near neighbors (because the strong force saturates, as stated above). So to increase the force binding the protons together we have to add more neutrons. So eventually, N becomes much larger than Z. We can observe how this would be true if we consider a line of protons – say n protons in a row, which are connected by rope which can have a tension only of Ts where Ts is the strong nuclear force. As n gets larger, eventually the repulsive force on the last proton will be greater than Ts, and that is when the atom splits. A way to keep this from happening is to add more neutrons. Suppose we intersperse a proton and neutron in the chain. Then the furthest out proton experiences a smaller force b/c its much further away from the center of the other protons, but the adhesive force is the same. So it is more stable.

This works to a point, but as the nucleus grows in size, the imbalance of N over Z weakens the strong force (see 2nd point), and is further compromised by the fact that it can only act over a short distance (see 4th point), and so eventually the strong force can no longer hold the nucleus together in a stable arrangement. This is when nuclei typically decay (split) into smaller more stable nuclei.

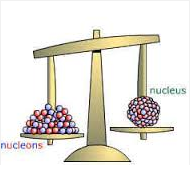
An estimate of the stability of a given nucleus is obtained by calculating its binding energy per nucleon. By analogy, recall that in the ground state an electron in a hydrogen atom has an energy of -13.6eV. Thus is would take 13.6 eV of energy to separate the electron from the proton. Or saying it backwards, you would have to take out 13.6eV from a separated electron and proton to bind them into a Hydrogen atom. So 13.6eV is the binding energy, B. This energy difference would show up in the mass of the constituent particles. Since E = mc2, we’d have:



FYI, Δm itself is called the mass defect. It works similar for a nucleus. Its binding energy is simply the energy that must be taken out to bind the the nucleons together, or stated oppositely, the energy required to separate the nucleons into individual nucleons. So B is the difference in mass between the nucleus, and the sum of the masses of all its constituent nucleons, times c2.



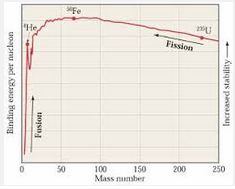
Since the binding energy is negative, you’ll note this implies that a nucleus is lighter than the total mass of its constituents.



Finally, the binding energy per nucleon, i.e. B/A, tells how tightly each nucleon is being held, and so this provides an estimate for the cohesiveness of the nucleus:



A plot of B/A is shown for the elements in the periodic table. As you can see, it drops once you get past roughly iron, indicating that heavier elements tend to be less and less stable.



Here’s a few things of note when calculating binding energies. First, a convenient mass unit for nucleons is the atomic mass unit (u). It is equal to 1/12 the mass of a carbon atom isotope 12C.



and is approximately the mass of a proton or neutron. The mass of an electron is 1u/1830 or so. The rest energy of 1u is E = 931.5 MeV. Second, the total mass of a stable nucleus, atom is always less than the mass of its constituents since the mass is a measure of the energy and the energy consists of the rest energy of the constituents + the negative binding potential energy (from strong nuclear force). Third, as a technical note, one can usually neglect the binding energies of electrons as they pertain to mass. For instance, the binding energy of the e- in 1H in the ground state is 13.6 eV, which is much less than ~ MeV.

Should throw this out there too. So when talking about binding energies, we’re talking about the difference in energy between ground state configurations. But similar to the Bohr model of the e- in an atom, the nuclei themselves have excited states. As a rough estimate, using the particle-in-box model, since nucleons are confined to distance of around 10fm, and have mass around m = 1.67×10-27kg, their energy level spacing would be around: E = h2/8mL2 = h2/8mp(10fm)2 ~ 1 MeV, compared ~ 10eV for electrons in an atom.

**Example**

Let’s calculate the binding energy (or *mass defect*) of deuterium, 2H. This would be,



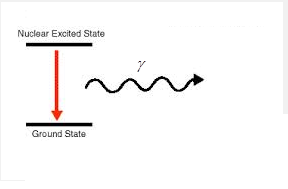
Note how in the second line we combined the p and e- into 1H, neglecting its binding energy. Typical binding energies are 6 – 9 MeV per nucleon.

**Nuclear Reactions**

Just as molecules can undergo chemical reactions, nuclei can undergo nuclear reactions. Some require a net input of energy to get them to proceed, and some release energy. The former are called endothermic reactions, while the latter are called exothermic reactions. The mass defect, Δm, will distinguish these two. Endothermic reactions have a positive mass defect, while exothermic reactions have a negative mass defect. Relatedly, the energy absorbed or released is designated Q = -(Δm)c2, and is negative for endothermic reactions, and positive for exothermic reactions. Guess I’ll look at a few common reactions.

***Gamma rays***

When a nucleus drops from one excited state to another it will emit a photon (just like electrons do) called a γ ray in this context, with an energy equal to the energy level spacing.



This process is symbolized like this:



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***β particles***

β particles are really just electrons. When the number of neutrons in a nucleus becomes too large to keep the nucleus cohesive, one or more neutrons will often decay into a proton and an electron (the β particle), releasing some energy in the process too. The process is represented below,



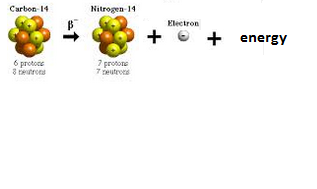
Of course we can reverse the process too.



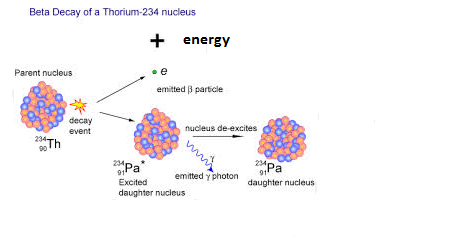
As far as the nucleus is concerned, this process would be represented as:



where Y is the element with one more proton than X. The energy usually shows up as kinetic energy of the ‘daughter nucleus’ Y, or the β particle. Most of the energy usually goes to the lighter particle, i.e. the β particle.

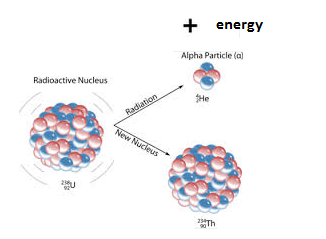


After emission of these fragments, a nucleus is often left in an excited state, so that instead of , we would have: . Then the excited state decays to the ground state via emission of a gamma ray (as in the previous discussion).



***α particles***

When the number of protons gets too large for the nucleus, it often emits an α particle. An α particle is just a He nucleus, i.e. 2 protons + 2 neutrons.



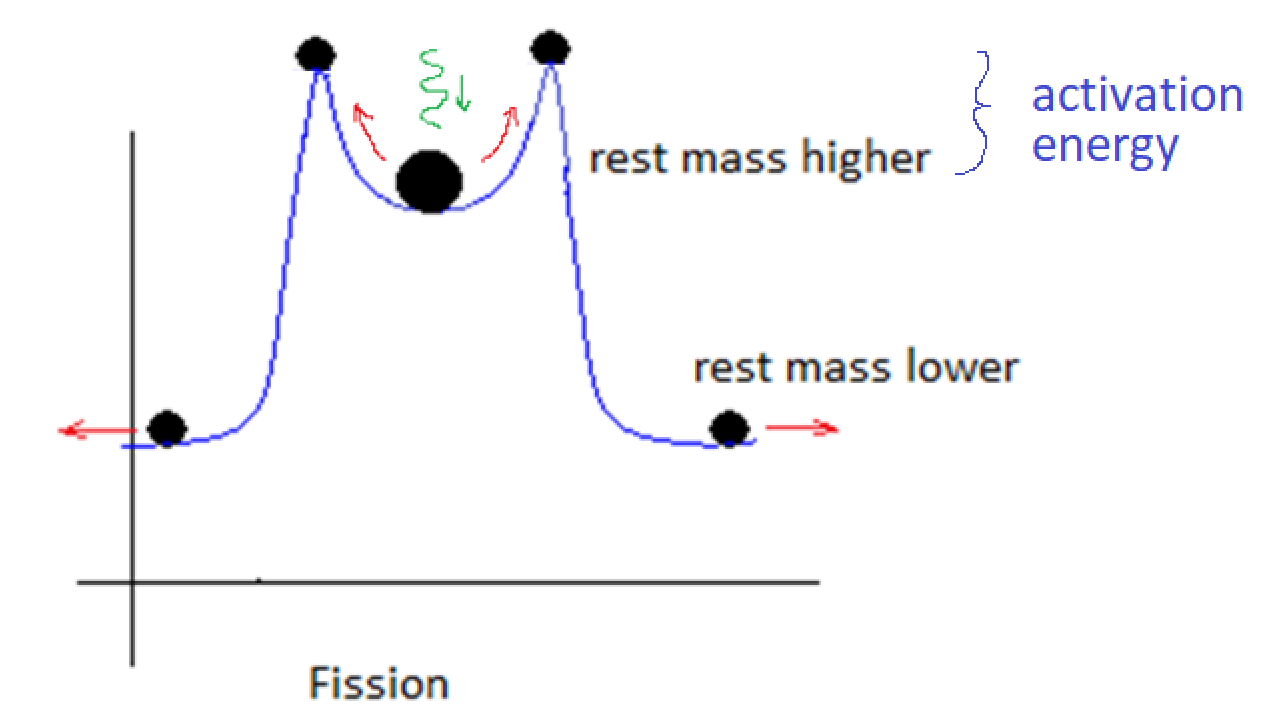
The notation would look like this:



where Y would be the nucleus with two less protons than X. The energy would again go to the ‘daughter nucleus’ Y, and the α particle. Most of it will go to the α since it is lightest.

**Nuclear Fission**

Some heavy elements’ isotopes can be induced to split: a process called *fission*. This will happen if the binding energy per nucleon is lower for the products than for the reactant. As we see in that B/A graph above, elements for which A >> 60 or so will satisfy this criteria. Often this requires a little energy input (sort of like the *activation* energy in chemistry) to get the process started since there is still a (weak) strong force attraction. Diagrammatically, it looks like this,

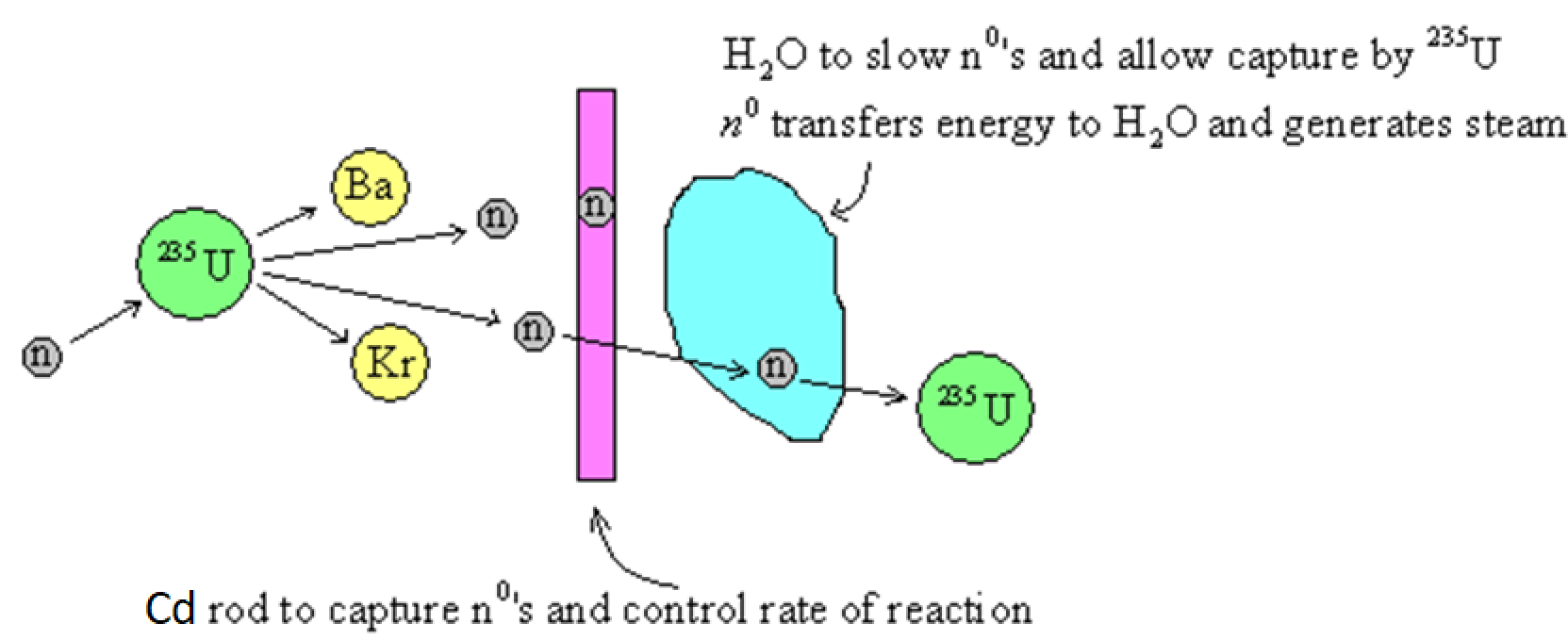


Being a little more mathematical about the energy conversions, we have:



In words….we start with a stable-ish nucleus with rest energy m12c2 and we send in a little KE perhaps in the form of a gamma wave. The rest energy can be broken down into the rest energies of the individual nuclei fragments that are going to split + their nuclear potential energy + their electric potential energy. In the second line, we’re saying that the KE input will enable the fragments to surmount the attractive PEn barrier (and due to increased separation, their PEe will change too). And then in the third line, the residual PE´e will be converted to kinetic energy as the electric force pushes the fragments apart. The net energy released would be Q = KE´ - KE = m12c2 – (m1+m2)c2. It’s possible this process could occur w/o the initial input KE, via quantum mechanical tunneling. But at the very least, the input KE would make the process more likely, and therefore more quickly occuring, as the PEn barrier the fragments have to tunnel through would be smaller.

Let’s consider Uranium fission. Uranium can be induced to split two smaller fragments of roughly equal size: Krypton and Barium. Both 235U (0.7% natural abundance) and 238U (99.3% natural abundance) can be induced to fission by bombarding them with neutrons. The more stable one, 238U requires neutrons with energies of at least 1.2 MeV to induce fission. On the other hand, the less stable, 235U needs slow moving neutrons, with energies less than 1eV. When fission is induced on either uranium, it will split into Ba and Kr, and about 2 neutrons as well, releasing about 200 MeV in the process. The neutrons released will not have the requisite 1.2 MeV to induce a chain reaction of fission of 238U though. On the otherhand, if neutrons bombard 235U, they will have too much energy. So nuclear reactors typically use what’s called a moderator (usually water), to slow the neutrons down, so that they will induce 235U to fission. Its usually desirable to control the rate of reaction so as not to blow up the place. Therefore we would need to reduce the number of neutrons, and this can be done by inserting cadmium rods. These absorb excess neutrons. The 235U fission process is displayed below.



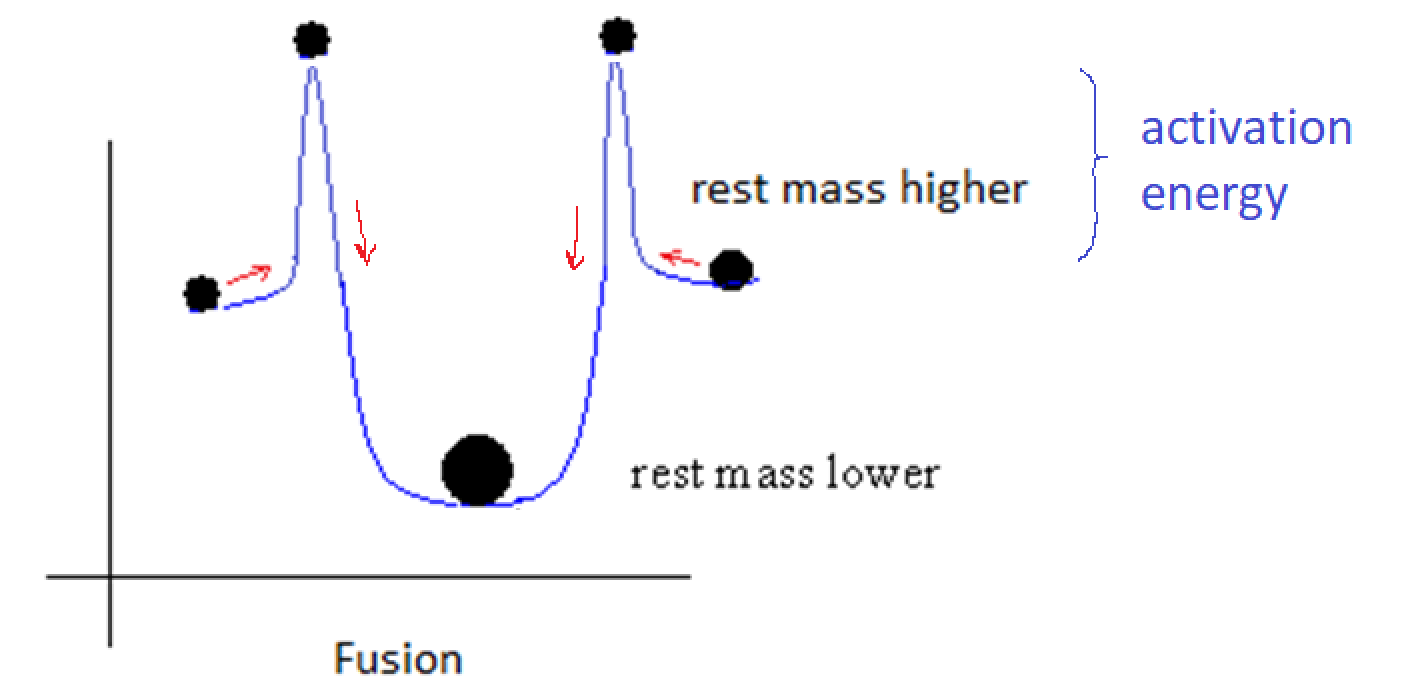
The process is 235U + n → 92Ba + 141Kr + 3n + energy. The binding energies per nucleon of 141Kr and 92Ba can be verified to be lower than 235U. The ‘activation’ energy is provided by the n in the LHS bombarding the 235U. As you can see, 3 neutrons are part of the products. These neutrons will absorb most the energy released, and will then impart this energy to nearby 235U’s, causing them to split too.

When one mines uranium, you get both isotopes. But as aforementioned, only about 0.7% of what you get is 235U. And you need to enrich it to about 3% 235U. The reason you need a certain percentage, I suppose, is that the 238U will absorb too many of the neutrons otherwise, and too few will be left for 235U to make the reaction self-sustaining.

The reaction takes place in water, which causes it to vaporize. The steam (thermal power) can then be used to run turbines, generating electricity. One of the hazards of nuclear decay is that the Ba, Kr will decay into more stable nuclei by β- emission, which itself is quite energetic. And there is no way to stop this. So one needs enough water to absorb the excess heat.

**Nuclear Fusion**

Nuclear fusion is another relevant process which can release a lot of energy. The binding energy per nucleon typically increases with A, up to about A = 60. So fusion of light nuclei into heavier nuclei would typically be an exothermic process. Often this requires a large ‘activation energy’, to get the process started. But once the reactants achieve this energy, they will coalesce into a more stable nucleus, and release a lot of energy in the process (I guess their potential energy at the top of the hill gets converted to kinetic energy as they roll to the bottom, and the maybe this kinetic energy is dissipated through photon emission or something). This is like what happens, I believe, during combustion of fuels, like gasoline and O2, or H2 in Cℓ. An initial bit of energy (like a spark) is needed to cause an initial combustion reaction, and then the large amount of energy given off makes the combustion of the rest of the reactants self-sustaining. Diagrammatically, the reaction looks like this:

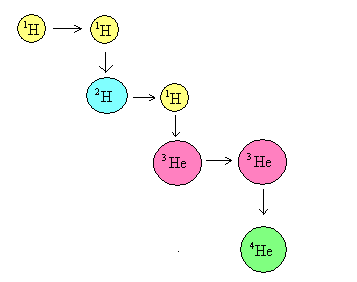


And mathematically, the energy conversion process would work out like this:



Which is to say, we start with two reactants and we give them kinetic energy. This kinetic energy must be enough to overcome the electric potential energy barrier, ie., we’ll presume all KE will be converted to PEe. Then the strong nuclear force will take over and bind the two nucleons together, resulting in (negative) nuclear potential energy PE´n, as well as increased electric potential energy PE´e, and presumably some residual KE´. The net energy difference is Q = KE´ - KE = (m1+m2)c2 – m12c2.

To illustrate, let’s consider a typical hydrogen fusion process. In pictures, it looks like this:



In other words,



So we get He out of deuterium (2H) basically. This is why it is so promising a technology, deuterium is all over the ocean. The Q value for this reaction is 26.7 MeV. In order for the reaction to occur, the two protons must get within a nuclear radius of each other which has a potential energy ~ 1MeV, as we saw before. Protons can have a KE typical of this value in very hot objects like the Sun, where typical T ~ 109 and therefore E ~ kT ~ 1MeV. No one has succeeded in creating a controlled fusion reactor. Typical methodologies are to try to laser heat a pellet of Hydrogen to the typical thermal energy above so that fusion can be initiated. I think this has been done. But the problem is then focusing the energy released to initiate fusion of nearby H’s I guess. Well, in a controlled way that doesn’t result in an explosion.

Fusion of H into He happens in the Sun via the reaction: 2H + 3H → 4He + n + energy. As can verify the binding energy per nucleon is smaller for 4He than for 2H and 3H. Deep in its core, its immense temperature, combined with the commensurate pressure provide the ‘activation energy’ to cause the two H’s to coalesce into a 4He, releasing a large amount of energy in the process – some of which prevents the Sun from collapsing under its own gravitational force, and some of which we receive as sunlight.

**Example**

Let’s consider the following process, where a Li isotope is bombarded by a proton, resulting in two α particles. So we have,



We can ignore the binding energies of the electrons again. The only important thing is to have the same number of electrons on each side. We get,



Therefore this reaction gives off energy; it is exothermic. Nonetheless, it does require energy to initiate it. This is because the proton 1H has to get inside the Li nucleus to begin the decay process. But the proton will be repelled by the +3*e* charge of the Li nucleus. The potential energy required would be:



So once the protons are given this energy, it can then cause the reaction. So roughly speaking, the net energy released would be Q – U = 16.11MeV. Well, we have to conserve momentum too and so it might be that we need more energy – should go to com reference frame.